A mathematical approach to model fatigue patterns in dynamic tasks

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Background

Fatigue progresses continuously and a binary classifier falls short in capturing its dynamics. In this study, a mathematical model is introduced that is capable of explaining the fatigue process across multiple work and rest cycles, based on the subjects' initial energy and a simple linear transformation of their ratings of perceived exertion (RPE). The model is defined based on the recurrence relations between the work cycles. Using this simple yet applicable approach, a unique function can be defined for each work cycle which depends on its previous cycles. The model fitting, achieved high values of R2 close to 95% and even for more scattered experimental data, collected within a short period of time due to fatigue, R^2 of greater than 87% was achieved.

Experiment

Data was collected from a simulated dynamic upper shoulder order picking task lab experiment. Four combinations of bottle load and picking pace were chosen for four different lab sessions: 2.5 kg-5 bpm (bottle per minute), 2.5 kg- 10 bpm, 2.5kg - 5 bpm, and1.5 kg-15 bpm. RPE scores (on a scale of 0-10) were reported every 5 minutes by participants. Each session included three 45-minute work periods and two 15-minute rest breaks in between. A total of 14 participants (7 male) participated in this experiment.

Method

A recurrence model evaluated energy dynamics from the start of the task to its end, with consideration of energy decay rate (β) and rest-induced energy boosting rate (α) to explain the expected time to fatigue (t_2). The parameters were found through finding the fit with maximum R^2. Finally, the factors affecting the main parameters were analyzed using repeated measures and mixed-effects regressions.

Results

The goodness-of-fit analysis revealed that while some participant samples closely matched the model predictions (e.g., Sub06-1.5-15 with an R^2 of 95%), others displayed higher variance (e.g., Sub17-2.5-10 with an R^2 of 87%). The results of the repeated measure showed significant impact of pace on time-to-fatigue and significant impact of work periods (1 to 3) on both the time-to-fatigue and the energy boosting rate. Mixed effects model validated the significant impact of pace on time-to-fatigue. Moreover, it was found that as pace increases the time-to-fatigue decreases significantly. It is also shown that the energy boosting rate is significantly affected by the work period and energy decay rate is significantly affected by BMI and the highest level of pace (15 bpm). In conclusion, when modeling fatigue in dynamic tasks, practitioners need to consider the impact of time, task factors, and workers' characteristics on fatigue trajectories when designing tasks and rest breaks.